

Gov 50: 21. Hypothesis testing

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Roadmap

1. The lady tasting tea
2. Hypothesis tests
3. Hypothesis testing using infer

1/ The lady tasting tea

The lady tasting tea

Your friend asks you to grab a tea with milk for her before meeting up and she says that she prefers tea poured before the milk. You stop by a local tea shop and ask for a tea with milk. When you bring it to her, she complains that it was prepared milk-first.

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 - Prepare 8 cups of tea, 4 milk-first, 4 tea-first
 - Present cups to friend in a **random** order
 - Ask friend to pick which 4 of the 8 were milk-first.

Lady Tasting Tea data

Friend picks out all 4 milk-first cups correctly!

```
library(gov50data)
tea
```

```
## # A tibble: 8 x 2
##   truth      guess
##   <chr>      <chr>
## 1 tea-first  tea-first
## 2 milk-first milk-first
## 3 milk-first milk-first
## 4 tea-first  tea-first
## 5 tea-first  tea-first
## 6 milk-first milk-first
## 7 tea-first  tea-first
## 8 milk-first milk-first
```

Thought experiment

Could she have been guessing at random? What would guessing look like?

```
set.seed(02138)
one_guess <- tea |>
  mutate(random_guess = sample(guess))
one_guess
```

```
## # A tibble: 8 x 3
##   truth      guess      random_guess
##   <chr>     <chr>     <chr>
## 1 tea-first tea-first milk-first
## 2 milk-first milk-first tea-first
## 3 milk-first milk-first tea-first
## 4 tea-first tea-first milk-first
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## 6 milk-first milk-first milk-first
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```

4 correct in this random guess!

Another guess

```
another_guess <- tea |>
  mutate(random_guess = sample(guess))
another_guess
```

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## # A tibble: 8 x 3
##   truth      guess      random_guess
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## 6 milk-first milk-first  milk-first
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```

6 correct in this random guess!

All possible guesses

We could enumerate all possible guesses. “Guessing” would mean choosing one of these at random:

```
## Cup 1 Cup 2 Cup 3 Cup 4 Cup 5 Cup 6 Cup 7 Cup 8
## 1 milk milk milk milk tea tea tea tea
## 2 milk milk milk tea milk tea tea tea
## 3 milk milk tea milk milk tea tea tea
## 4 milk tea milk milk milk tea tea tea
## 5 tea milk milk milk milk tea tea tea
## 6 milk milk milk tea tea milk tea tea
```

[snip]

```
## Cup 1 Cup 2 Cup 3 Cup 4 Cup 5 Cup 6 Cup 7 Cup 8
## 65 tea tea tea milk milk tea milk milk
## 66 milk tea tea tea tea milk milk milk
## 67 tea milk tea tea tea milk milk milk
## 68 tea tea milk tea tea milk milk milk
## 69 tea tea tea milk tea milk milk milk
## 70 tea tea tea tea milk milk milk milk
```

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 - Impossible? No, because of random chance!

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 - Last YouGov poll of 1,363 likely voters said 44% planned to vote for Trump.
 - Could the difference between the poll and the outcome be just due to random chance?

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- **Probabilistic** proof by contradiction: try to “disprove” the null.

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- Data: poll has $\bar{X} = 0.44$ with $n = 1363$.

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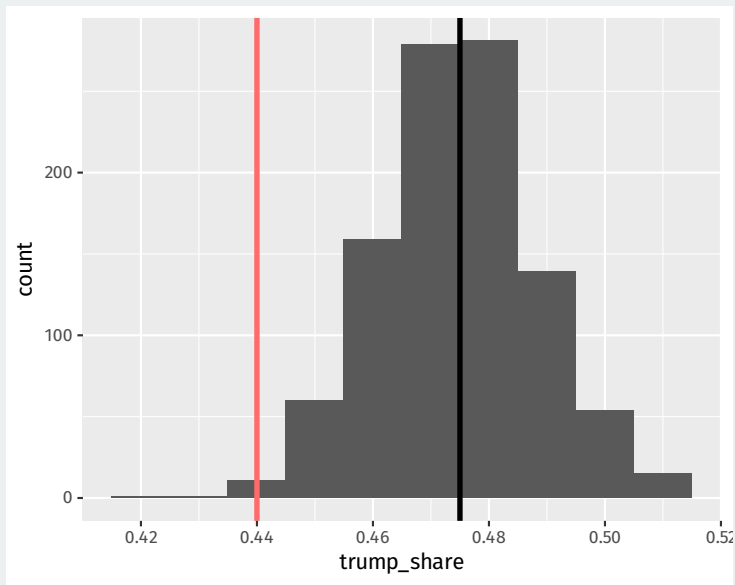
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- We can simulate sums of coin flips using a function called `rbinom()`
- Compare the distribution of proportions under the null to the observed proportion.

```
null_dist <- tibble(  
  trump_share = rbinom(n = 1000, size = 1363, prob = 0.475) / 1363  
)  
ggplot(null_dist, aes(x = trump_share)) +  
  geom_histogram(binwidth = 0.01) +  
  geom_vline(xintercept = 0.44, color = "indianred1", size = 1.25) +  
  geom_vline(xintercept = 0.475, size = 1.25)
```


Simulations of the reference distribution



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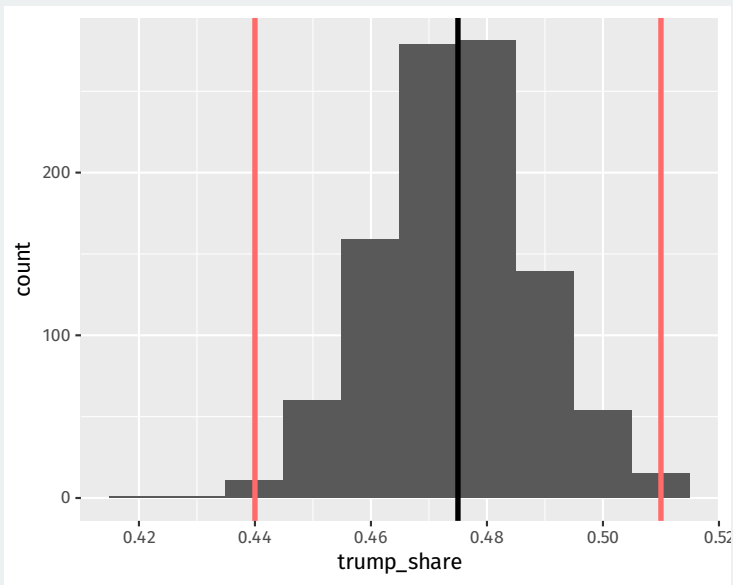
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 - Probability of sample proportions being greater than $0.475 + 0.035 = 0.51$.

```
mean(null_dist$trump_share < 0.44) + mean(null_dist$trump_share > 0.51)
```

```
## [1] 0.01
```

Two-sided p-value



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 - $p < 0.05$ “statistically significant”
 - $p < 0.01$ “highly significant”

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- Type I error because it's the worst
 - “Convicting” an innocent null hypothesis
- Type II error less serious
 - Missed out on an awesome finding

3/ Hypothesis testing using infer

GSS data from infer

```
library(infer)
```

```
gss
```

```
## # A tibble: 500 x 11
```

```
##   year  age sex  college partyid hompop hours income
##   <dbl> <dbl> <fct> <fct>    <fct>    <dbl> <dbl> <ord>
## 1  2014   36 male  degree   ind         3    50 $25000~
## 2  1994   34 female no degree rep         4    31 $20000~
## 3  1998   24 male  degree   ind         1    40 $25000~
## 4  1996   42 male  no degree ind         4    40 $25000~
## 5  1994   31 male  degree   rep         2    40 $25000~
## 6  1996   32 female no degree rep         4    53 $25000~
## 7  1990   48 female no degree dem         2    32 $25000~
## 8  2016   36 female degree   ind         1    20 $25000~
## 9  2000   30 female degree   rep         5    40 $25000~
## 10 1998   33 female no degree dem         2    40 $15000~
## # i 490 more rows
## # i 3 more variables: class <fct>, finrela <fct>,
## #   weight <dbl>
```

What is the average hours worked?

dplyr way:

```
gss |>
  summarize(mean(hours))
```

```
## # A tibble: 1 x 1
##   `mean(hours)`
##         <dbl>
## 1           41.4
```

infer way:

```
observed_mean <- gss |>
  specify(response = hours) |>
  calculate(stat = "mean")
observed_mean
```

```
## Response: hours (numeric)
## # A tibble: 1 x 1
##   stat
##   <dbl>
## 1  41.4
```


Hypothesis test

Could we get a mean this different from 40 hours if that was the true population average of hours worked?

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Null and alternative:

H_0 : population average hours = 40

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Null and alternative:

H_0 : population average hours = 40

H_1 : population average hours \neq 40

How do we perform this test using infer? The **bootstrap!**

Specifying the hypotheses

```
gss |>  
  specify(response = hours) |>  
  hypothesize(null = "point", mu = 40)
```

```
## Response: hours (numeric)  
## Null Hypothesis: point  
## # A tibble: 500 x 1  
##   hours  
##   <dbl>  
## 1     50  
## 2     31  
## 3     40  
## 4     40  
## 5     40  
## 6     53  
## 7     32  
## 8     20  
## 9     40  
## 10    40  
## # i 490 more rows
```

Generating the null distribution

We can use the bootstrap to determine how much variation there will be around 40 in the null distribution.

```
null_dist <- gss |>
  specify(response = hours) |>
  hypothesize(null = "point", mu = 40) |>
  generate(reps = 1000, type = "bootstrap") |>
  calculate(stat = "mean")
null_dist
```

```
## Response: hours (numeric)
## Null Hypothesis: point
## # A tibble: 1,000 x 2
##   replicate stat
##   <int> <dbl>
## 1         1  40.3
## 2         2  39.8
## 3         3  40.0
## 4         4  39.2
## 5         5  40.3
## 6         6  40.2
## 7         7  40.4
```

Visualizing the p-value

We can visualize our bootstrapped null distribution and the p-value as a shaded region:

```
null_dist |>
  visualize() +
  shade_p_value(observed_mean,
                direction = "two-sided")
```

Simulation-Based Null Distribution

