# Gov 50: 24. More Inference with Mathematical Models

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# Roadmap

1. Confidence intervals for experiments
2. Hypothesis testing with the CLT
3. Two-sample tests

# 1/ Confidence intervals for experiments

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- Estimated **average treatment effect**

$$\widehat{\text{ATE}} = \overline{Y}_T - \overline{Y}_C = 0.07$$

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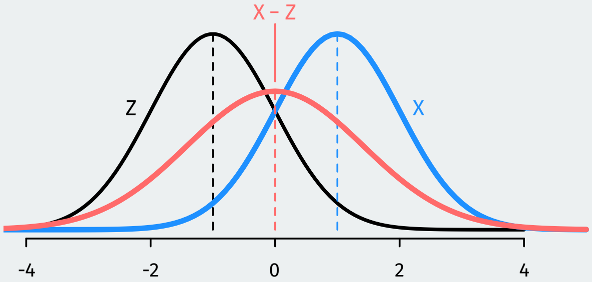
But what is the $SE_{\text{diff}}$ in this case?

# Spread of a difference in normals

If we take the difference between two independent normal r.v.s, what happens to the spread?




The spread of the difference is **larger** than the spread of the two variables being differenced!

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    - We can construct a 95% CI with $\widehat{\text{ATE}} \pm 1.96 \times SE_{\text{diff}}$

# Confidence intervals

But we don't know $\mu_T$ or $\mu_C$! Plug in our sample proportions to estimate the SE:

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Range of possibilities taking into account plausible chance errors.

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    5. Use p-value to decide whether to reject the null hypothesis or not

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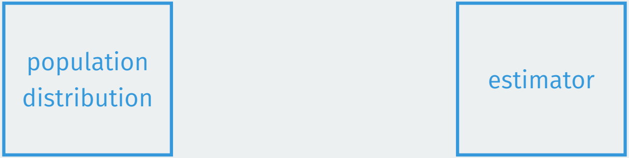

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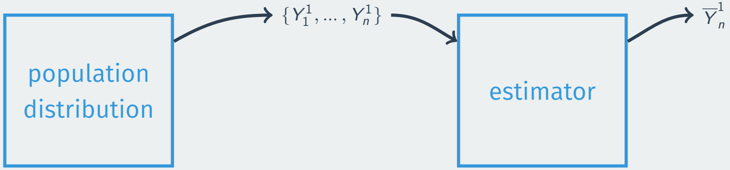

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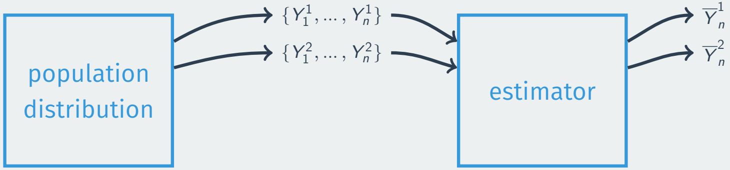

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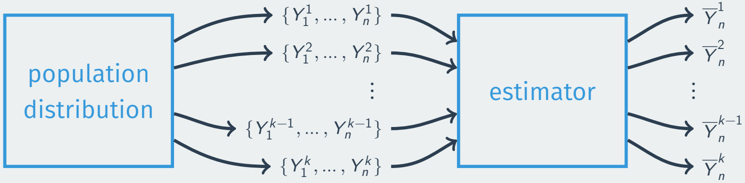

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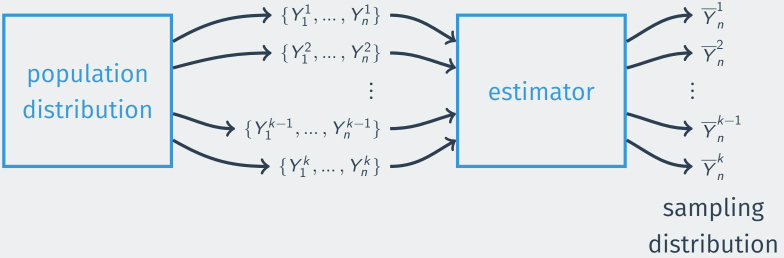

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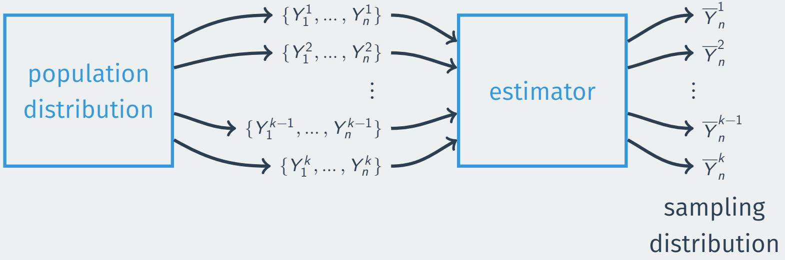

# CLT for hypothesis testing

Under the null, we know the distribution of $\overline{Y}$:

$$\overline{Y} \approx N\left(p, \frac{p(1-p)}{n}\right) = N\left(0.5, \frac{0.5 \times 0.5}{812}\right)$$

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Then under the null, know the distribution of the following test statistic:

$$Z = \frac{\overline{Y} - 0.5}{0.5/\sqrt{812}} \approx N(0, 1)$$

# p-values

What we observe:

$$Z_{\text{obs}} = \frac{\overline{Y} - 0.5}{0.5/\sqrt{812}} = \frac{0.42 - 0.5}{0.5/\sqrt{812}}$$

$$= -\frac{0.08}{0.018} = -4.44$$

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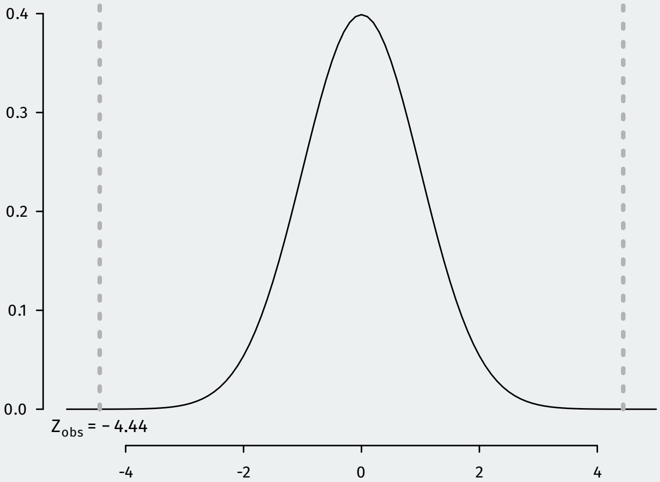
Our observed sample proportion is 4.44 SEs away from 0.5 under the null. What's the probability of being that far away? (**p-value**)

```
pnorm(-4.44, mean = 0, sd = 1) + ## prob being below -4.44
  (1 - pnorm(4.44, mean = 0, sd = 1)) ## prob being above 4.44
```

```
## [1] 0.000009
```


0.4
0.3
0.2
0.1
0.0
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-4
-2
0
2
4

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- p-value: probability of a standard normal being bigger than $|Z_{obs}|$

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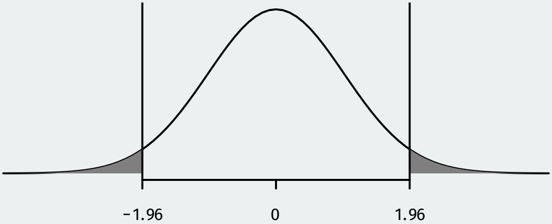


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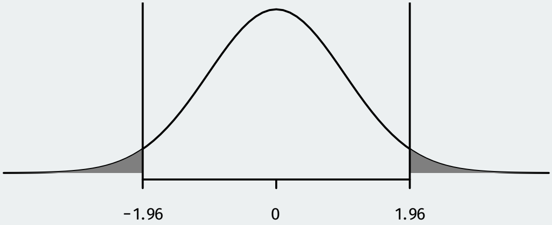


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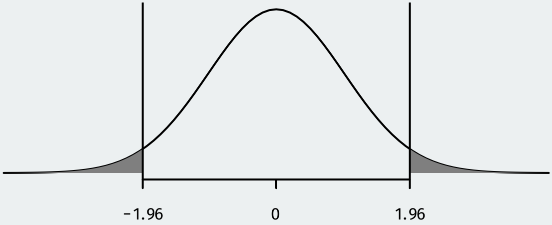


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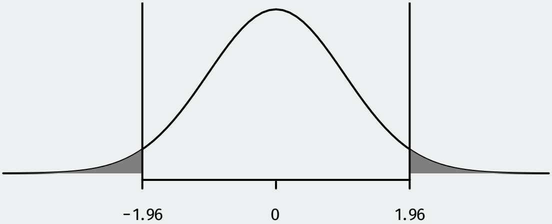


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    - We can reject if $|Z_{\text{obs}}| > 1.96$

# **3/** Two-sample tests

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    - Null: $H_0 : \mu_T - \mu_C = 0$
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# Two-sample hypotheses

- Parameter: **population ATE** $\mu_T - \mu_C$
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    - Null: $H_0 : \mu_T - \mu_C = 0$
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- In words: are the differences in sample means just due to chance?

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- Estimated SE for the difference in means:

$$\widehat{\text{SE}}_{\text{diff}} = \sqrt{\frac{\overline{Y}_T(1-\overline{Y}_T)}{n_T} + \frac{\overline{Y}_C(1-\overline{Y}_C)}{n_C}} = 0.028$$

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Same general form of the test statistic as with one sample mean/proportion:

$$\frac{\text{observed - null guess}}{\text{SE}}$$

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- In large samples, we can replace true SE with an estimate:

$$\widehat{\text{SE}}_{\text{diff}} = \sqrt{\widehat{\text{SE}}_T^2 + \widehat{\text{SE}}_C^2}$$

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- Finally! Our test statistic in this sample:

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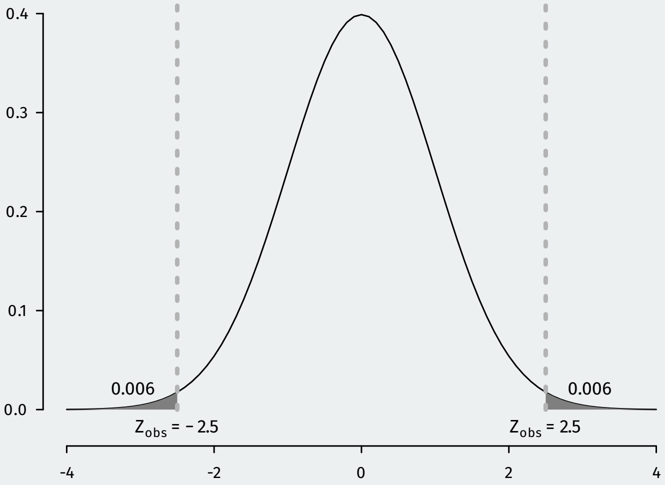
- p-value based on a two-sided test: probability of getting a difference in means this big (or bigger) if the null hypothesis were true
    - Lower p-values $\rightsquigarrow$ stronger evidence against the null.




```r
2 * pnorm(2.5, lower.tail = FALSE)
```

```
## [1] 0.0124
```